Accurately Approximate Formula of Effective Filling Fraction for Microstrip Line with Isotropic Substrate and Its Application to the Case with Anisotropic Substrate

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***Abstract*—The accurately approximate formula of effective filling fraction q_w is obtained for the microstrip line with zero-thickness strip and isotropic substrate. The line capacitance per unit length C/ϵ_0 for the case with anisotropic substrate can easily be obtained by using the approximate formula of q_w and C_0/ϵ_0 tabulated for the wide range of shape ratio w/h. The parameters Z, v, and λ for such a line can be calculated by using C/ϵ_0 and C_0/ϵ_0.**

I. Introduction

THE CALCULATION of the parameters of a microstrip line based on a TEM approximation is useful for the design of microwave integrated-circuit structure. The parameters can be derived from the line capacitances. However, the line capacitances may change from zero to infinity in accordance with the shape ratio w/h. This makes it difficult to derive the accurately approximate formula of the parameters for an arbitrary microstrip line. For example, the approximations for characteristic impedance have been obtained by Wheeler [1], [2]. Wheeler has proposed the effective filling fraction q_w in [1], where w is added as a suffix after Wheeler to differentiate the effective filling fraction q and the electric charge q. Subsequently, he has shown the approximation of q_w for isotropic dielectric substrate in [2]. This q_w depends mainly on the shape ratio and only slightly on the relative dielectric constant of substrate. Also, it has the desired properties that $q_w \to 0.5$ when $w/h \to 0$, and $q_w \to 1$ when $w/h \to \infty$. The effective relative dielectric constant ϵ^*_{eff} can be simply computed from $\epsilon^*_{\text{eff}} = 1 + q_w(\epsilon^* - 1)$. One of the authors has derived the Green's function for the microstrip line with an anisotropic substrate [3] and has proposed, by using this Green's function, a method which calculates with a high accuracy the line capacitance of such a line. This method also calculates accurately q_w for the isotropic substrate. Recently, we have shown that the electrostatic problem with multilayered anisotropic media can be transformed into that with multilayered isotropic media [4]. We have found from the fact of this transformation that the line capacitance per unit length for the microstrip line with anisotropic dielectric substrate is equal to that for the microstrip line with a corresponding isotropic dielectric substrate.

In this paper, we show the accurately approximate formula of q_w for the microstrip line with isotropic substrate. We show that C/ϵ_0 and the parameters Z, v, and λ, for the case with anisotropic substrate, can be obtained by using the approximate formula of q_w and C_0/ϵ_0 tabulated in this paper for the wide range of shape ratio w/h.

II. Effective Filling Fraction and Its Application

The parameters of a microstrip line can be calculated by using the line capacitances per unit length C/ϵ_0 of the case with substrate and C_0/ϵ_0 of the case without substrate. These C/ϵ_0 and C_0/ϵ_0 can be calculated with a high accuracy by using the Green's function technique ([3], after the next corrections, $(--)v \to (--)^{\gamma}$ in (37) and (46)). Also, the exact value of C_0/ϵ_0 can be obtained by conformal mapping as shown in the Appendix. We consider the microstrip line where its strip width is w and the permittivity tensor of its anisotropic dielectric substrate of thickness h is such that

$$\bar{\epsilon} = \begin{bmatrix} \epsilon^*_\xi & 0 \\ 0 & \epsilon^*_\eta \end{bmatrix} \epsilon_0 \tag{1}$$

in the ξ–η coordinates obtained by rotating the x–y coordinates with the angle γ. The x–y axes are parallel and perpendicular to the ground conductor, respectively. As having mentioned in [4], this microstrip line can be transformed into the microstrip line with w', h', and isotropic dielectric substrate ϵ' as follows:

$$w' = w \tag{2}$$

$$h' = \alpha h / \{(\alpha^2 - 1)\cos^2\gamma + 1\} \tag{3}$$

$$\epsilon' = \epsilon^* \epsilon_0 \quad \epsilon^* = \sqrt{\epsilon^*_\xi \epsilon^*_\eta} \tag{4}$$

where

$$\alpha = \sqrt{\epsilon^*_\eta / \epsilon^*_\xi}\ . \tag{5}$$

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TABLE I
VALUES OF WH_i AND $q_w(WH_i, \epsilon^*)$

ε*	WH_i	$q_w(WH_i, \epsilon^*)$
1.01	4.1	0.774848
2.00	4.2	0.767128
8.00	4.4	0.761774

TABLE II
VALUES OF b_1, b_2, d_1, d_2, d_3, AND d_4

ε*	b_1	b_2	d_1	d_2	d_3	d_4
1.01	-0.9914970	-0.3383140	0.8129886	-0.2530581	-0.2495215	-0.04892659
2.00	-0.9999744	-0.3375214	0.8727037	-0.2856383	-0.2803518	-0.05496000
8.00	-1.012575	-0.3487503	0.9281990	-0.3367463	-0.3189153	-0.06147304

The line capacitances per unit length for both microstrip lines are identical with each other [4]. This means that C/ϵ_0 for the microstrip line with the anisotropic substrate, as shown below, can easily be obtained from the approximate formula of q_w for the microstrip line with isotropic substrate if it is known. Therefore, it is worthy to obtain the approximate formula of q_w.

If we calculate q_w for the microstrip line with isotropic substrate ϵ^* and shape ratio w/h by the Green's function technique [3], we find that q_w for the zero-thickness strip can be approximated with the quadratic curve and with the high accuracy as follows:

$$q_w(WH, \epsilon^*) = a_0 + a_1 f(\epsilon^*) + a_2\{f(\epsilon^*)\}^2 \tag{6}$$

where $f(u) = 1 - 1/u$, $WH = w/h$, and the unknown coefficients a_0, a_1, and a_2 can be given by solving the simultaneous equations obtained by substituting q_w for $\epsilon^* = 1.01$, 2, and 8 in the case of same shape ratio to the considered one, WH, into (6). Therefore, we can calculate the value of (6) if q_w for $\epsilon^* = 1.01$, 2, and 8 is known for an arbitrary shape ratio. We can obtain the approximation of q_w for $\epsilon^* = 1.01$, 2, and 8 by using the line capacitance for such microstrip lines calculated by the Green's function technique [3]. That is,

$$q_w(WH, \epsilon^*) = \begin{cases} 1 - \{1 - q_w(WH_i, \epsilon^*)\} \exp\left[b_1 \log_{10}(WH/WH_i) + b_2\{\log_{10}(WH/WH_i)\}^2\right], & WH_i \leq WH \quad (7) \\ 0.5 - \{0.5 - q_w(WH_i, \epsilon^*)\} \exp\left[d_1 \log_{10}(WH/WH_i) + d_2\{\log_{10}(WH/WH_i)\}^2 \right. \\ \left. \quad + d_3\{\log_{10}(WH/WH_i)\}^3 + d_4\{\log_{10}(WH/WH_i)\}^4\right], & WH_i \geq WH. \quad (8) \end{cases}$$

In (7) and (8) the values of WH_i and $q_w(WH_i, \epsilon^*)$, for $\epsilon^* = 1.01$, 2, and 8 are shown in Table I, and the values of b_1, b_2, d_1, d_2, d_3, and d_4 are shown in Table II. In Table III, the percentage error of the approximate formula (6) of the effective filling fraction (method B) to the results calculated by the Green's function technique [3] (method A) are compared with those calculated by the authors using Wheeler's formulas (eq. (53), [2]) (method W1) and (eq. (59), [2]) (method W2). We calculated the characteristic impedance for the microstrip line without dielectric substrate (R_1) in the Wheeler's formulas from $R_1 = (4\pi v_0 \times 10^{-12})/(C_0/\epsilon_0)$ by using C_0/ϵ_0 shown in Table IV and the velocity of light, $v_0 = 2.997925 \times 10^8$ m/s. Method W1 has a good accuracy for the narrow strips as having been mentioned by Wheeler [2]. Method W2 has a good accuracy for the intermediate and wide strips. We can understand that the relative error of method B is less than 0.2 percent since method A has a very high accuracy, as already mentioned in the previous paper [3]. This, the approximate formula (6), has a high accuracy. However, the demerit of this formula is that it is not simpler than the Wheeler's formulas.

Next, we consider the microstrip line with w/h and anisotropic dielectric substrate $\bar{\epsilon}$ as shown in (1). The line capacitance per unit length C/ϵ_0 and the parameters Z, v, and λ of this microstrip line for zero-thickness strip can be calculated by using (2)–(6) and Table IV as follows:

$$C/\epsilon_0 = C'/\epsilon_0 \tag{9}$$

$$Z/Z_c = 1/\left\{(C_0/\epsilon_0)\sqrt{\epsilon^*_{\text{eff}}}\right\} \tag{10}$$

$$v/v_0 = \lambda/\lambda_0 = 1/\sqrt{\epsilon^*_{\text{eff}}} \tag{11}$$

where

$$C'/\epsilon_0 = \epsilon^{*\prime}_{\text{eff}} C'_0/\epsilon_0 \tag{12}$$

$$\epsilon^*_{\text{eff}} = (C/\epsilon_0)/(C_0/\epsilon_0) \tag{13}$$

$$\epsilon^{*\prime}_{\text{eff}} = 1 + (\epsilon^* - 1) q_w(w'/h', \epsilon^*) \tag{14}$$

$Z_c = \sqrt{\mu_0/\epsilon_0}$ intrinsic impedance of the free space,
$C/\epsilon_0, C_0/\epsilon_0$ line capacitances per unit length for the microstrip line with w/h and $\bar{\epsilon}$,
$C'/\epsilon_0, C'_0/\epsilon_0$ line capacitances per unit length for the microstrip line with w'/h' and ϵ^*,

TABLE III
PERCENTAGE ERRORS OF THE APPROXIMATIONS OF THE EFFECTIVE FILLING FRACTION q_w TO METHOD A FOR THE MICROSTRIP LINE

w/h \ ε*		1.01	1.5	2	4	8	16	128
0.01	W1	0.004	0.27	-0.37	-0.44	-0.47	-0.46	-0.53
	W2	-2.0	-1.9	-1.8	-1.5	-1.4	-1.3	-1.3
	B	-0.094	-0.12	-0.11	-0.049	-0.029	-0.002	-0.047
0.1	W1	-0.005	-0.53	-0.75	-0.98	-1.0	-1.0	-1.0
	W2	-2.3	-2.2	-2.1	-1.8	-1.6	-1.4	-1.3
	B	-0.008	-0.076	-0.088	-0.099	-0.090	-0.079	-0.067
1	W1	0.73	-0.90	-1.6	-2.7	-3.1	-3.3	-3.5
	W2	-0.68	-0.66	-0.56	-0.26	-0.026	0.12	0.27
	B	0.097	0.11	0.097	0.087	0.11	0.13	0.15
10	W2	-0.52	-0.59	-0.59	-0.53	-0.47	-0.42	-0.38
	B	0.012	0.024	0.019	0.016	0.024	0.032	0.042
100	W2	-0.27	-0.27	-0.22	-0.24	-0.23	-0.22	-0.21
	B	0.075	0.052	0.095	0.067	0.083	0.094	0.11

W1: method by the approximate formula[2,(53)]
W2: method by the approximate formula[2,(59)]
B : method by the approximate formula(6)

TABLE IV
EXACT LINE CAPACITANCE PER UNIT LENGTH (C_0/ϵ_0) OF THE MICROSTRIP WITHOUT SUBSTRATE

a \ w/h	a	2a	3a	4a	5a	6a	7a	8a	9a
0.0001	0.556537	0.592941	0.616532	0.634441	0.649066	0.661525	0.672439	0.682188	0.691025
0.001	0.699126	0.757553	0.796490	0.826636	0.851638	0.873217	0.892334	0.909583	0.925362
0.01	0.939947	1.04869	1.12480	1.18587	1.23800	1.28412	1.32588	1.36431	1.40010
0.1	1.43375	1.70270	1.91198	2.09392	2.25993	2.41548	2.56364	2.70633	2.84479
1	2.97989	4.23155	5.39883	6.52694	7.63141	8.71990	9.79678	10.8648	11.9259
10	12.9813	23.362	33.596	43.766	53.899	64.010	74.103	84.185	94.257
100	104.32	204.75	305.00	405.18	505.32	605.44	705.54	805.62	905.70
1000	1005.7	2006.2	3006.4	4006.6	5006.8	6006.9	7007.0	8007.1	9007.2

computed by the authors using conformal mapping [5], [6] for w/h ≤ 10 and using the Green's function technique [3] for w/h > 10

q_w effective filling fraction for the microstrip line with w'/h' and ϵ^*.

w', h', and ϵ^* are denoted in (2), (3), and (4), respectively.

III. CONCLUSION

We have proposed the accurately approximate formula of the effective filling fraction for the microstrip line with isotropic substrate. The relative error of this formula is less than 0.2 percent by comparing with the results obtained by the Green's function technique. It has been shown by applying the transformation of the anisotropic problem into the isotropic problem, that the line capacitance per unit length C/ϵ_0 and the parameters Z, v, and λ of the microstrip line with anisotropic substrate can easily be calculated by using the approximate formula of the effective filling fraction for the case of isotropic substrate. The numerical calculation has been performed with the aid of the digital computers HITAC 8250 and HITAC 8800/8700.

APPENDIX

The line capacitance C_0/ϵ_0 per unit length of the microstrip line without a substrate can be found analytically by conformal mapping [5], [6], and by the Green's function technique [3]. It is useful in using the approximate formula (6) that the C_0/ϵ_0 is tabulated for the wide range of shape ratio. The results calculated by the authors are shown in Table IV. All values shown omitted the following results: the results for the shape ratio $w/h \leqslant 10$ which are calculated by conformal mapping; the results for $w/h > 10$ which are calculated by the Green's function technique, because of the overflow in the digital computer while it was used to calculate the conformal mapping.

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